Column Bending Test Analysis

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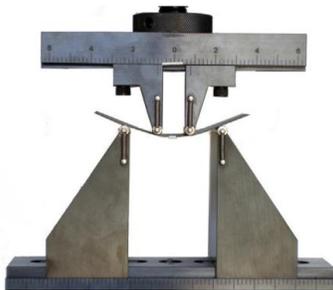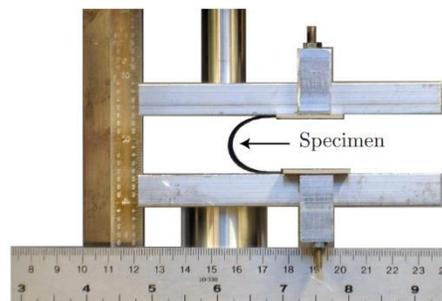
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Methods of analysing the moment curvature behaviour of thin woven composites

There are several experimental methods to study the moment curvature behavior and failure of high strain composites such as,
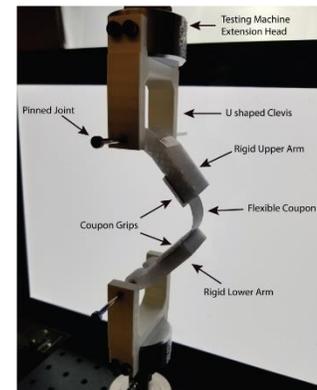
1. Four point bending test
2. Platen folding test
3. Column bending test

four point bending test

platen folding test

Column bending test

Limitations associated with some techniques

The first two methods inherit several limitations in observing the complete and accurate moment-curvature behavior of ultra-thin composites due to their design, as listed below:

- **Four point bending test** – This method can be used to observe the initial moment-curvature response. However, it is limited by the occurrence of large elastic deformations in thin woven composites, leading to slippage from the supports.
- **Platen folding test** - Moment distribution throughout the specimen is highly non-uniform leading large errors in processing of results. Pure moments are not acting at point of transition between flat and curved sections. Transverse loads induce compressive strains at coupon mid-section

Background of CBT

The main objective of Column Bending Test is to obtain the moment versus curvature behaviour in the geometrically nonlinear regime for various high strain composites. The slope of moment versus curvature curve provides the value of bending stiffness (D11) for a particular choice of layup as a function of the applied curvature, revealing possible material non-linearities. The test also provides the failure curvature of the material, which is one of the main parameters necessary for the design of deployable space structures using HSC.

This work analyzes a 2019 Caltech CBT experiment using three methods. First, a closed-form geometric analysis is employed. Then, two corrections are incorporated into the simple geometric analysis, resulting in two additional methods. Finally, the results from all three methods are plotted alongside the results from a numerical analysis, four-point bending test, and platen folding test failure data.

1. Closed form solution

In a conventional CBT experiment, which is done by using a universal testing machine, the outputs are the incremental deflection and the applied force readings.

With the recent advancements in digital image correlation techniques, most of the CBT experiments are done with 3D DIC systems to read direct curvature values of the specimen.

In this analysis moment curvature response of the specimen is obtained by analysing the applied force and displacement readings using the simple geometric analysis, which utilize several assumptions in order to simplify the problem.

Assumptions used in the analysis

1. Neutral axis always remains at the centre of the specimen during bending

2. Bending moment, and therefore the curvature, is close to constant along the arc length (If the coupon is sufficiently shorter compared to the length of the arms, the variation of the bending moment within the specimen is small)

3. Length of the specimen remains constant during the test, since the compressive loading is neglible compared to the bending moment

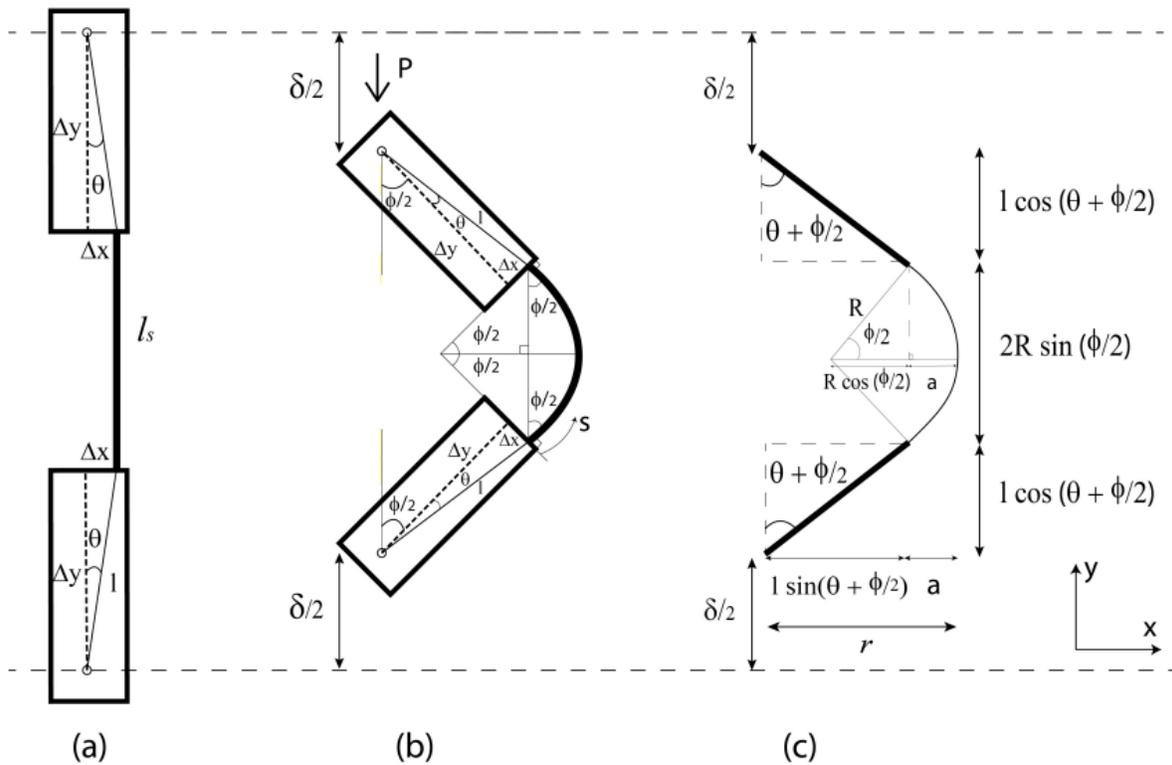4. Specimen leaves the rigid arm at a perfect 90° angle

Fig. 3 Geometry of the Column Bending Test: (a) Initial test geometry, (b) Angles and (c) Distances in the test setup when a displacement δ is applied.

Figure courtesy: Sharma et al., 2019

l_s - free length of the samples between the rigid arms or gauge length (assumed to be constant during the test),

l - effective length of the rigid fixture arm (i.e., the distance between sample and pin)

θ - initial angle of the fixture arm,

φ - change in fixture arm angle due to deflection during the test

δ - linear vertical displacement of the fixture

P - represents the load applied by the testing machine to the fixture

r - effective moment arm length,

a - horizontal component of the distance between center and end of the coupon

R - represents the radius of curvature of the bent coupon.

Initial vertical distance,

$$d_{initial} = 2l \cos(\theta) + l_s$$

After applying a displacement δ which results in an arm rotation of $\phi / 2$

$$d_{final} = \delta + 2R \sin(\phi / 2) + 2l \cos(\theta + \phi / 2)$$

Applying $d_{initial} = d_{final}$,

$$\frac{\delta}{l_s} = 1 - \frac{2}{\phi} \sin \frac{\phi}{2} + 2 \frac{l}{l_s} \left(\cos \theta - \cos \left(\theta + \frac{\phi}{2} \right) \right)$$

The above equation for ϕ is transcendental and requires a numerical solution. We use a MATLAB numerical solution to solve this problem in a latter part of this study.

As we have obtained the arm rotation, we could calculate the curvature of the specimen using $l_s = R\phi$, $\kappa = 1/R$

$$\kappa = \frac{\phi}{l_s}$$

Then the calculations are done to obtain the moment at each curvature. Calculation of effective moment arm length to obtain **M**$_{max}$

$$r = a + l\sin\left(\theta + \frac{\phi}{2}\right)$$

$$a = R - R\cos\left(\frac{\phi}{2}\right)$$

Substituting for **a**, **R** and dividing by **s** we can obtain an expression for r,

$$\frac{r}{s} = \frac{1}{\phi}\left(1 - \cos\frac{\phi}{2}\right) + \frac{l}{s}\sin\left(\theta + \frac{\phi}{2}\right)$$

$$M_{max} = Pr$$

$$M_{min} = Pl\sin\left(\theta + \frac{\phi}{2}\right)$$

2. <u>Non linear constitutive model for analysis of CBT</u>

Murphey et.al. developed a non-linear fiber based constitutive model to describe the behavior of thin flexures subjected to large strains.

Only the fibre is assumed to be non-linear.

Fibre modulus according to this model;

$$E_{fT}(\varepsilon) = E_o(1 + \gamma_T\varepsilon) \text{ for } \varepsilon \geq 0 \ (Fibres \ in \ tension)$$
$$E_{fC}(\varepsilon) = E_o(1 + \gamma_C\varepsilon) \text{ for } \varepsilon < 0 \ (Fibres \ in \ compression)$$

γ_T, γ_C — Tensile and compressive fiber non-linear parameters

E_o — Initial fiber modulus at zero strain.

Using the rule of mixtures,

$$E_c = E_f V_f + E_m(1 - V_f)$$

V_f – fibre volume fraction

E_c – Composite modulii

E_n – Resin modulii

E_f – Fibre modulii

According to Murphey et al, there are two composite modulii for tension and compression. Which results in stiffening under tension and softening in compression.

$$E_{cT} = E_o(1 + \gamma_T \varepsilon)V_f + E_m(1 - V_f) \text{ for } \varepsilon \geq 0$$
$$E_{cC} = E_o(1 + \gamma_C \varepsilon)V_f + E_m(1 - V_f) \text{ for } \varepsilon < 0$$

Assumptions to simplify the constitutive model

1 . $E_1 \gg E_2$; that neglect the contribution of E_2 in expression for D_{11}

2. $\gamma_T = \gamma_C = \gamma$.

3. $E_C = E_f$

Above assumption lead to the formation of single expression

$$E_f(\varepsilon) = E_o(1 + \gamma\varepsilon)$$

Slope of the stress-strain is the fibre modulus. Hence integration of the fibre modulus w.r.t strain provides the express for the stress.
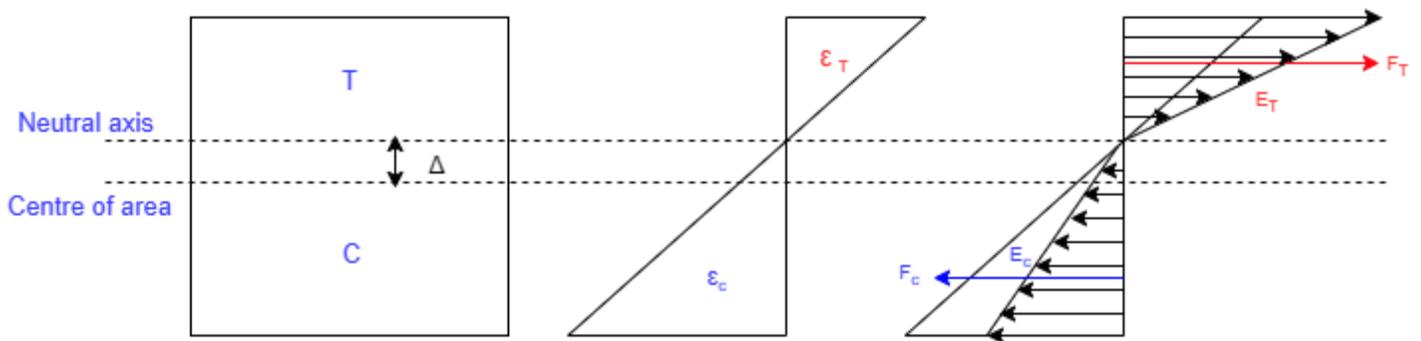
$$\sigma_c = \int E_o (1 + \gamma\varepsilon) \, d\varepsilon$$

$$\sigma_c = E_o\varepsilon + \frac{E_o\gamma\varepsilon^2}{2}$$

This tensile stiffening and compression softening, results in shifting of neutral axis towards the tension side of the specimen under bending,

Results in higher strains on the compression side with respect to the tension side. This phenomenon is explained in the following figure.

Considering the equilibrium of the section,

$F_T = F_C$

But, $E_T > E_C$

Hence, $\varepsilon_c > \varepsilon_T$ at similar distances from the center of area of the cross section

$\therefore \Delta > 0$; shift of the neutral axis towards the tension zone

According to this model there is a critical compressive strain where instantaneous modulus becomes zero.

$$\varepsilon_{c,cr} = -\frac{1}{\gamma}$$

After this strain, tensile stress presents at compressive strains hence solution is unphysical.

Expression for the moment curvature response using the non-linear constituent model

Considering the force equilibrium through the thickness of the specimen

$$N_c = \int_{-t/2}^{t/2} \sigma_c dy = 0$$

Here stress is a function of strain. Hence it is required to establish the strain as a function of the thickness co-ordinate. Using Euler-Bernoulli theory,

$$\varepsilon = \kappa(y - y_{na})$$

By substituting strain equation in the stress equation and integrating over the thickness we get an expression for the neutral axis.

$$y_{na} = \frac{6 - \sqrt{36 - 3t^2\gamma^2\kappa^2}}{6\gamma\kappa}$$

Moment of the specimen at a given curvature value can be expressed as below,

$$M = \int_{-t/2}^{t/2} \sigma_c(y - y_{na}) \, dy$$

Utilizing the expression for stress and the expression for the neutral axis, following expression can be constructed for the moment by integrating the above expression over the thickness of the specimen,

$$M = \frac{1}{72} E_1 \kappa t^3 \sqrt{36 - 3t^2\gamma^2\kappa^2}$$

E_1 is the initial linear stiffness of the composite along the direction of the fibers, can be approximated by $E_1 = E_0 V_f$ neglecting the contribution of the matrix.

D_{11} is the bending stiffness of the material. Which is the slope of the moment-curvature function.

$$D_{11} = \frac{\partial M}{\partial \kappa} = \frac{E_1 t^3 (6 - t^2\gamma^2\kappa^2)}{12\sqrt{36 - 3t^2\gamma^2\kappa^2}}$$

3. <u>Theory of Elastica for the analysis</u>

If the hypothesis of constant curvature along the arc is not true, the behavior of the coupon needs to be modeled using Euler's theory of elastica, which accounts for large scale deflection of 1D structural elements.

<u>Assumptions</u>

1. The loading axis coincides with the specimen axis hence $\Delta x = 0$

utilize the previous constitutive equation

$$M(s) = \frac{1}{72} E_1 t^3 \kappa(s) \sqrt{36 - 3t^2 \gamma^2 \kappa(s)^2} = \frac{1}{72} E_1 t^3 \beta'(s) \sqrt{36 - 3t^2 \gamma^2 \beta'(s)^2}$$

In here, moment and curvatures are functions of the location of the arc, and $\beta^{\cdot}$ is the angle between the a vector tangent to the coupon and the vertical axis at every point along the curve.

differential equations for the position co-ordinates x(s) and y(s) can be given in terms of the angle $\beta(s)$

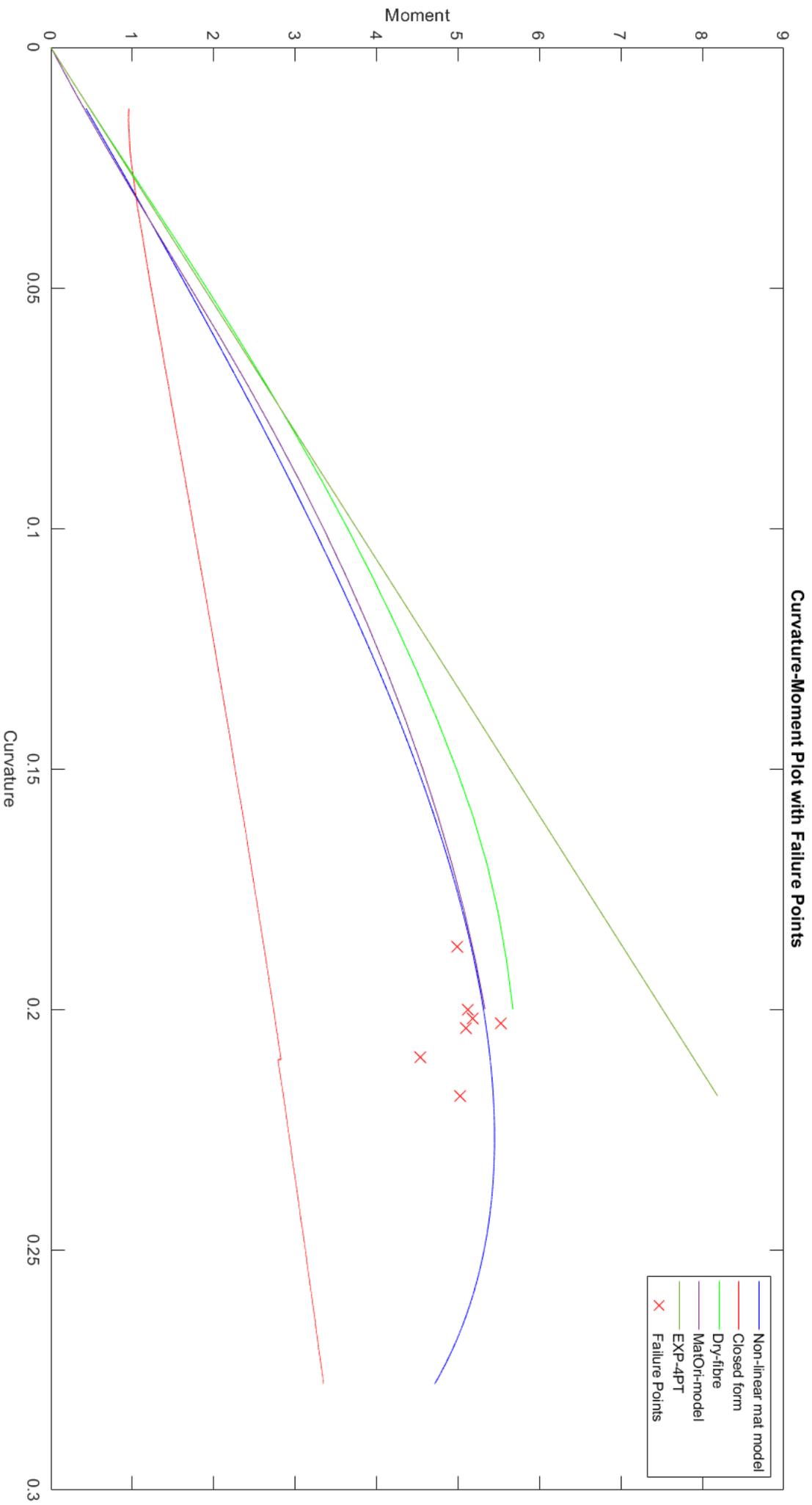$$\frac{dx}{ds} = \sin \beta$$
$$\frac{dy}{ds} = \cos \beta$$

For the geometry of the problem, equilibrium of bending moment gives,

$$\frac{1}{72} E_1 t^3 \frac{d\beta}{ds} \sqrt{36 - 3t^2\gamma^2 \left(\frac{d\beta}{ds}\right)^2} = M = P\left(x + l\sin\frac{\phi}{2}\right)$$
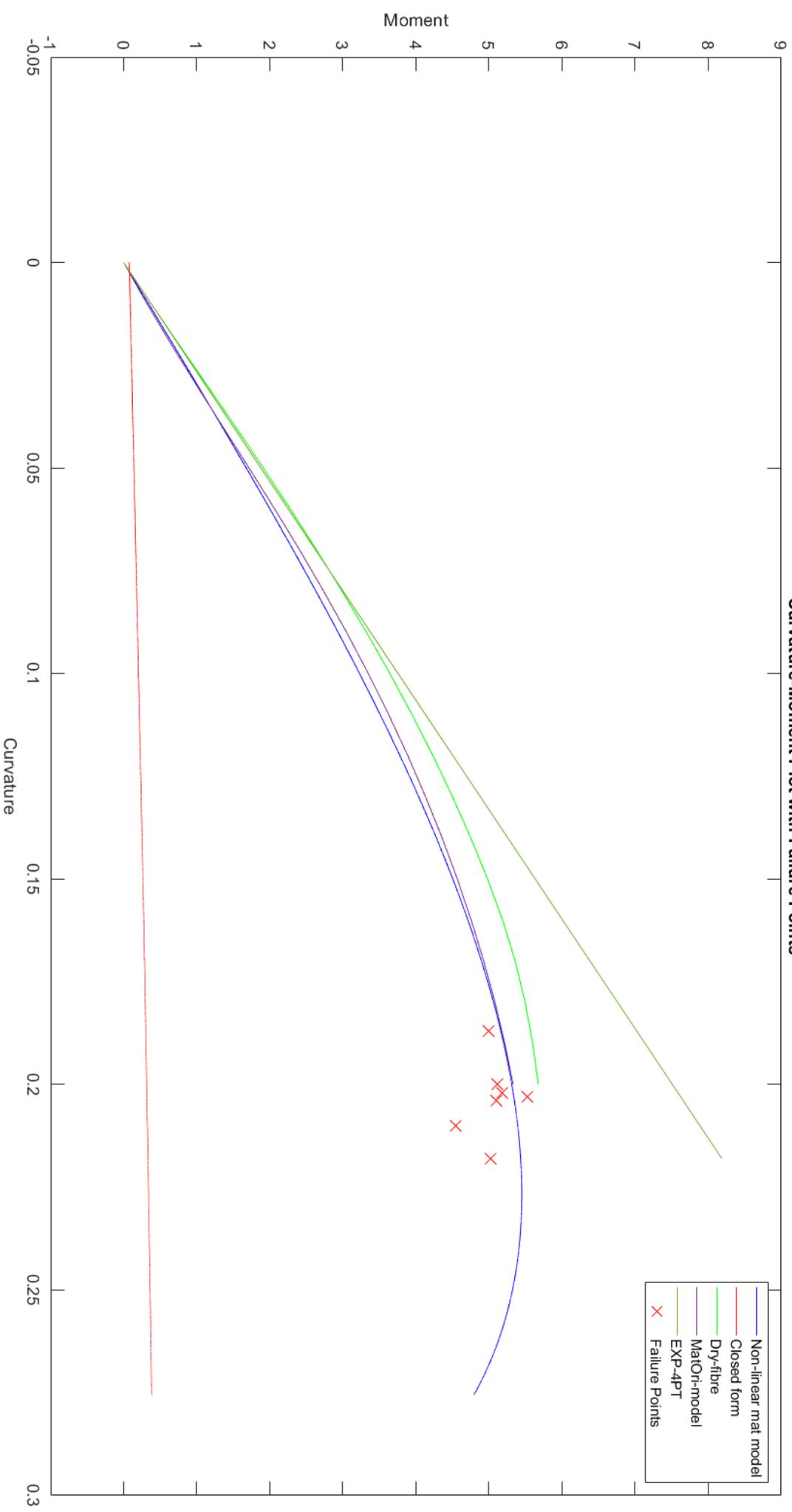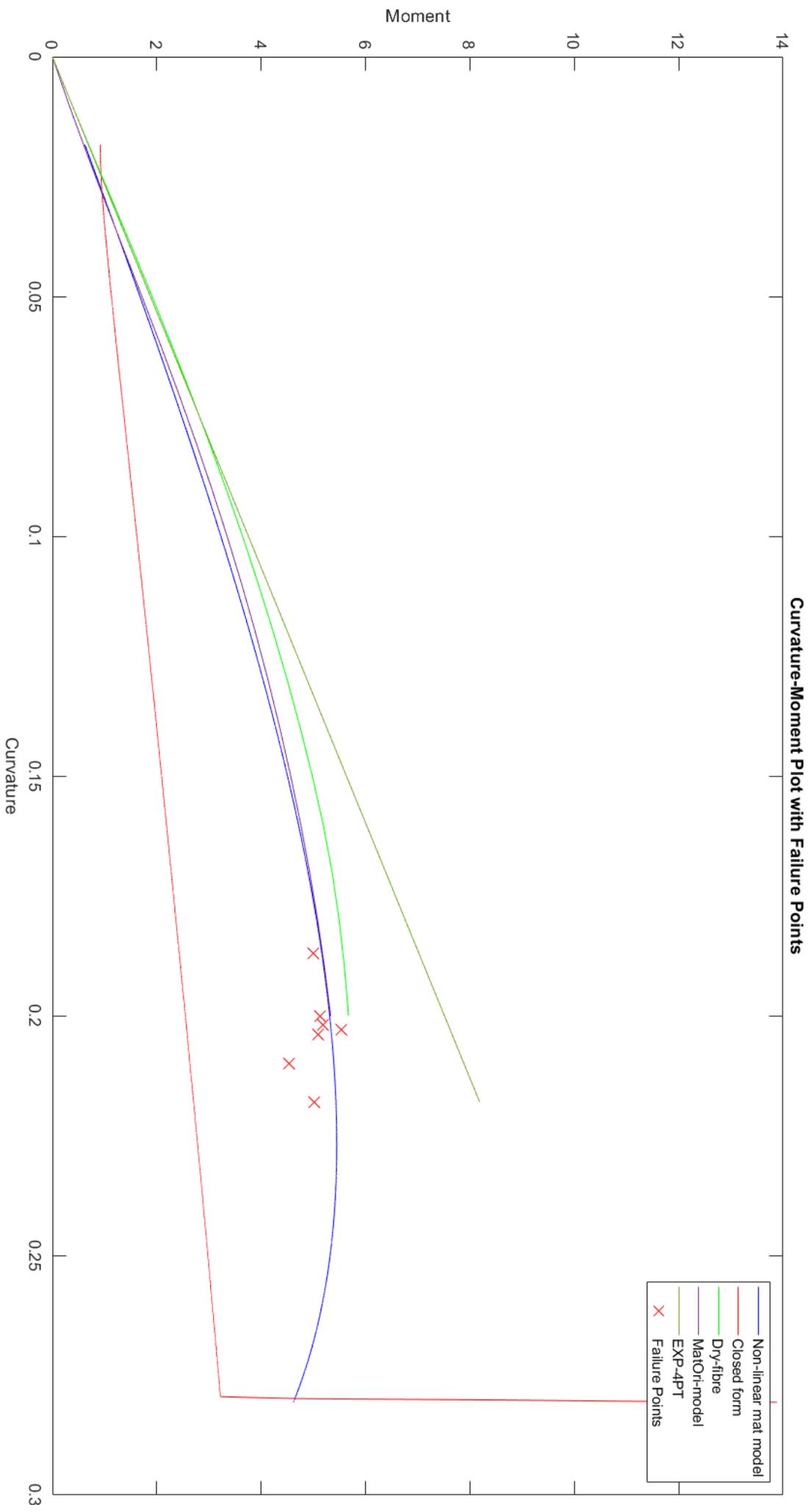
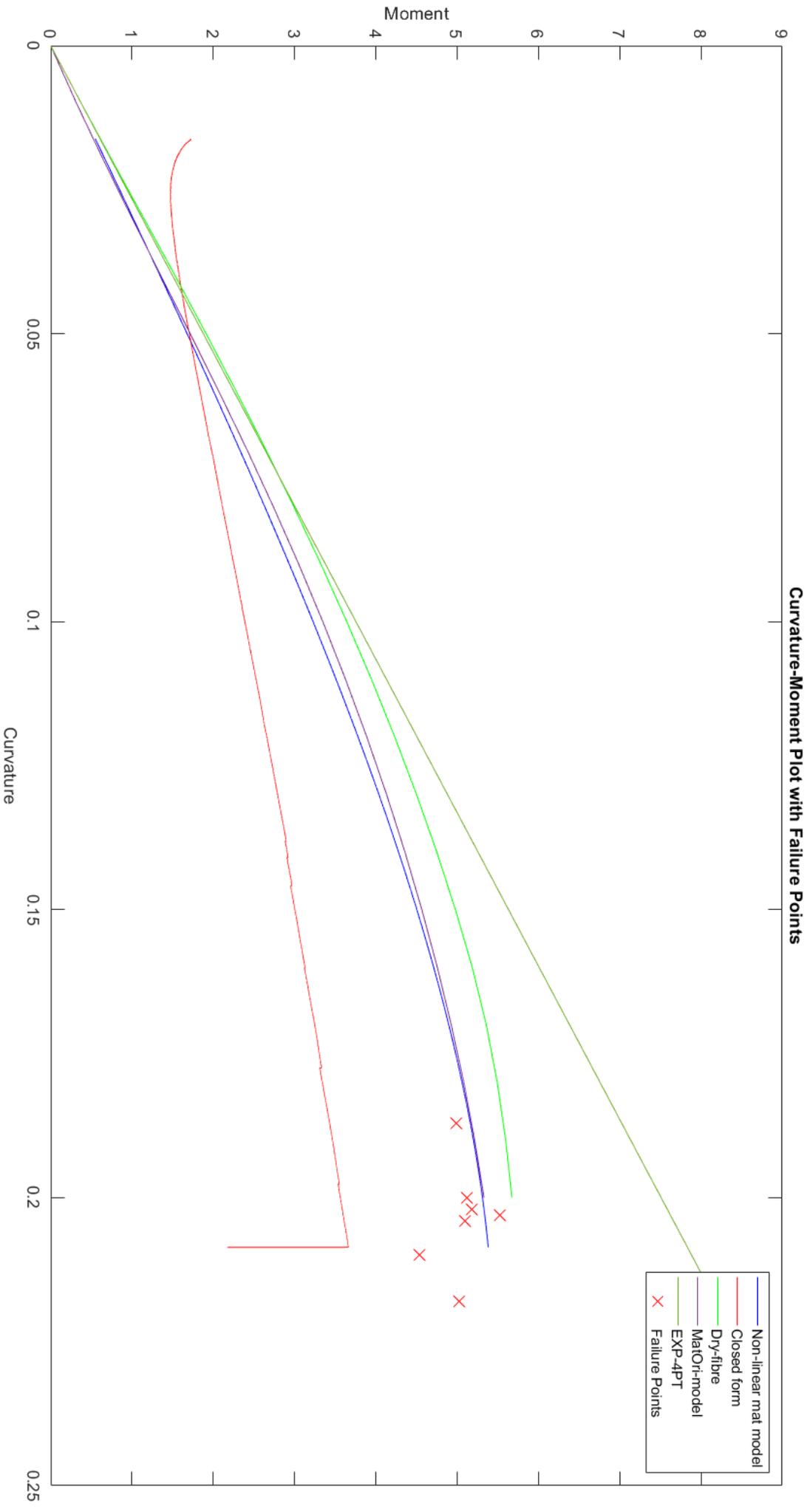## Results

Parameters used for the analysis

- Free length = 10 mm
- Width of the specimen = 20.33 mm
- Real arm length = 28 mm
- Initial deviation of the loading axis = 4.7 mm
- Fibre volume fraction = 0.62
- Thickness of the specimen = 0.18 mm
- E_1 of fibres = 233000 (N/mm^2)
- Factor of material non-linearity = 60

Curvature-Moment Plot with Failure Points

Curvature-Moment Plot with Failure Points

Curvature-Moment Plot with Failure Points

Curvature-Moment Plot with Failure Points

End